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STABILITY OF A CYLINDRICAL SHELL IN A STRATIFIED FLOW*

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Analysis of the Kelvin stability of a horizontal plane flow of two fluid layers of different densities and one moving relative to the other is extended to the case of longitudinal coaxial flow of a two-layered fluid within a circular cylindrical shell. It is shown that the loss of stability of the whole system sets in for low velocities of the layer motion, one relative to the other. A comparison is made with classical flutter for which the flow is not stratified.

1. We consider the motion of a thin, elastic, infinitely long circular cylindrical shell subjected to perturbations of an internal longitudinal potential flow of a two-layered ideal incompressible fluid. The layer interface has the shape of a coaxial circular cylindrical surface in the unperturbed state. The fluid flow perturbations are considered to be sufficiently small so that their squares and higher powers can be neglected. Equations of the bending theory of shells are used neglecting tangential inertial forces /1/.

By virtue of the axial symmetry of the shell-fluid system, we separate the circumferential coordinate and we obtain for the n-th circumferential mode of the potentials of the perturbed layer velocities and the shell displacements

$$\varphi_{xx}^s + \varphi_{rr}^s + r^{-1}\varphi_r^s - n^2r^{-2}\varphi^s = 0 \tag{1.1}$$

$$\Delta^2\Delta^2\Phi + \frac{(1-\nu^2)R_1^4}{a^2}\Phi_{xxxx} + \frac{\rho h R_1^4}{D}\Delta^2\Phi_{tt} = \frac{R_1^4}{D}P^1\Big|_{r=R_1} \tag{1.2}$$

$$\Delta\Phi = R_1^2\Phi_{xx} - n^2\Phi, \quad D = \frac{Eh^3}{12(1-\nu^2)}, \quad a = \frac{h}{\sqrt{12}R_1}$$

$$P^s = -\rho_0^s(\varphi_t^s + V_s\varphi_x^s), \quad W^1 = \Delta^2\Phi \tag{1.3}$$

$$r = R_s, \ \varphi_r^1 = W_t^s + V_1W_x^s \tag{1.4}$$

$$r = R_2, \ \varphi_r^2 = W_t^2 + V_2W_x^2, \ P^1 = P^2$$

$$r = 0, \ |\varphi^2| < \infty; \ 0 < R_2 < R_1, \ s = 1, 2$$

Here x and r are the longitudinal and radial coordinates of a cylindrical coordinate system coupled to the axis of the shell-fluid system φ^s, P^s is the potential of the velocity perturbations and the appropriate pressure for the outer $(s=1)$ and inner $(s=2)$ fluid layers, R_1, h, ρ are the radius, thickness, and density of the shell material, D is the cylindrical stiffness, a is the shell thin-walledness parameter, ρ_0^s, V_s are the fluid layer density and velocity for unperturbed motion, W^s are the shell normal displacements $(s=1, r=R_1)$ and the layer interface boundary $(s=2, r=R_2)$ respectively.

2. We shall seek the solution of the problem for the shell, the potential φ^s, and the pressure P^s in the class of longitudinally propagating waves

$$(W^s, \Phi, \varphi^s, P^s) = (W_+^s, \Phi_+, \varphi_+^s(r), P_+^s(r))\, e^{i(\omega t - kx)} \tag{2.1}$$

Here ω is the vibration frequency $k=\pi/\lambda$ is the wave number, λ is the half-wavelength in the generatrix direction, and W_+^s, Φ_+, $\varphi_+^s(r), P_+^s(r)$ are coefficients and functions to be determined.

Substitution of (2.1) into (1.1) and conditions (1.4) yields a boundary value problem whose solution we seek in the form

$$\varphi_+^1(r) = C_1^1I_n(kr) + C_2^1K_n(kr), \ \varphi_+^2(r) = C^2I_n(kr) \tag{2.2}$$

Eqs.(1.1) and the last condition in (1.4) are satisfied here and we have from the first two conditions in (1.4) (V is the phase velocity of wave propagation)

$$C_1^1I_n'(kR_s) + C_2^1K_n'(kR_s) = i(V - V_1)W^s \tag{2.3}$$

$$C^2I_n'(kR_2) = i(V - V_2)W^2, \ V = \omega/k$$

Here $I_n(x)$ and $K_n(x)$ are modified Bessel functions and Macdonald functions, respectively, and the plane denotes the derivative with respect to the argument.

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Determining the unknown C_j^1 and C^2 from (2.3) and using the first relationship of (1.3), taking (2.1) into account, we obtain expressions for the pressure

$$P_+^1(r) = -\rho_0^1 k (V - V_1)^2 [F(R_2, r) W^1 - F(R_1, r) W^2]$$
$$P_+^2(r) = \rho_0^2 k (V - V_2)^2 (I_n(kr)/I_n'(kR_2)) W^2 \tag{2.4}$$
$$F(x, y) = \frac{I_n'(kx) K_n(ky) - I_n(ky) K_n'(kx)}{I_n'(kR_1) K_n'(kR_2) - I_n'(kR_2) K_n'(kR_1)}$$

Then substituting these expressions into (1.2) and the penultimate condition (1.4) by using (2.1), and assuming W^s to be non-zero, we obtain the equation

$$a_{11}a_{22} - a_{12}a_{21} = 0 \tag{2.5}$$
$$a_{11} = V_{kn}^2 - V^2 + \frac{\rho_0^1 (V - V_1)^2}{\rho k h} F(R_2, R_1)$$
$$a_{12} = -\frac{\rho_0^1 (V - V_1)^2}{\rho k h} F(R_1, R_1)$$
$$a_{21} = -\rho_0^1 (V - V_1)^2 F(R_2, R_2)$$
$$a_{22} = \rho_0^1 (V - V_1)^2 F(R_1, R_2) - \rho_0^2 (V - V_2)^2 I_n(kR_2)/I_n'(kR_2)$$
$$V_{kn}^2 = \frac{D}{\rho h R_1^2}\left(\delta^2 + \frac{1 - \nu^2}{a^2\delta^2}\right), \quad \delta^2 = \frac{(k^2R_1^2 + n^2)^2}{k^2R_1^2}$$

Here $V_{kn}(kR_1, n)$ is the phase velocity of the propagation of primarily bending waves along the shell in a vacuum with a minimum /1/ equal to $[2D\sqrt{1 - \nu^2}/(a\rho h R_1^2)]^{1/2}$.

3. We examine the case of the propagation of waves with a low phase velocity along the shell-fluid system for small velocities of layer motion, i.e.

$$(V/V_{kn})^2 \ll 1, \quad (V_s/V_{kn})^2 \ll 1 \tag{3.1}$$

By using the estimates (3.1), Eq.(2.5) is reduced to the form $a_{22} = 0$, which corresponds to the solution of the problem on fluid layer motion in a cylindrical cavity with solid walls. We hence find

$$V = \frac{V_1 - V_2\gamma_{kn}}{1 - \gamma_{kn}}, \quad \gamma_{kn} = \pm\left[\frac{\rho_0^2 I_n(kR_2)}{\rho_0^1 F(R_1, R_2) I_n'(kR_2)}\right]^{1/2} \tag{3.2}$$

It follows from the properties of the functions $I_n(x)$ and $K_n(x)$ and their derivatives /2/ that γ_{kn} is purely imaginary. Analysis of (3.2) shows that a phase velocity V with negative imaginary part exists for a two-layered stream $V_1 \neq V_2$, i.e., there is a travelling wave with a progressing amplitude in the shell-fluid system. In other words, three is already a loss of system stability for low velocities V_s of the layers, and the initial perturbations grow exponentially with time.

If $V_1 = V_2$, then the two-layered flow is transformed into a flow with constant velocity over a section and $V = V_1 = V_2$ is a real quantity, i.e., there is no exponential growth of the amplitudes for the low stream velocities under consideration ($V \ll V_{kn}$). The problem under investigation here reduces to the classical flutter problem. The solution of this problem for an infinite cylindrical shell within which there is a longitudinal potential fluid flow reduces to the magnitude of the critical flutter velocity V_* such that $V_* > V_{kn}$. As we know, excitation of system selfoscillations occurs for the flow velocity V_* and the initial perturbations grow exponentially with time /1/. If the same shell is empty, and an infinite fluid surrounds it by a coaxial two-layered flow, then by performing analogous calculations we obtain an expression for γ_{kn} that differs from that presented in (3.2) by the fact that the ratio $I_n(kR_2)/I_n'(kR_2)$ is replaced by $K_n(kR_2)/K_n'(kR_2)$ where $0 < R_1 < R_2 < \infty$. R_1, R_2 are the shell radii and the layer boundaries, respectively. Even in this case γ_{kn} is obviously purely imaginary and therefore, if $V_1 \neq V_2$ then the vibrations of the shell-fluid system grow exponentially with time for $V_s \ll V_{kn}$. If $V_1 = V_2$, then the system loss of stability sets in for the critical stream velocity such that $V_* > V_{kn}$ (flutter) /1/.

The special case $n = 0$ and $R_1, R_2 \to \infty$ such that $R_1 - R_2 = \text{const} = H$ will correspond to the solution of the plane problem for an infinite plate on one side of which a two-layered flow takes place. The near-wall layer here has the thickness H while the second layer is infinitely thick. Passing to the limit, we have $\gamma_{k0} = \pm i\sqrt{kH}$ $(kH \ll 1)$.

It is obvious that the presence of two layers in one of the fluids in a coaxial system of cylindrical shells and fluid flows also results in a loss of stability of the whole system for low flow velocities of the layers relative to each other. The system can here be in both a vacuum and in an infinite fluid.

Therefore, the analysis performed enables the following deduction to be made: the presence of a stratified fluid flow in a shell results in instability of the shell-fluid system starting with low layer velocities $(V_s \ll V_{kn})$. This system instability is generated by instability of the fluid layer boundary. If the fluid flow in the shell is not stratified, then as we know, the system loses stability for significantly higher stream velocities ($V_* > V_{kn}$, flutter).

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REFLECTION AND TRANSMISSION OF SOUND WAVES THROUGH THE INTERFACIAL BOUNDARY OF TWO JOINED ELASTIC HALF-STRIPS*

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A numerical solution of the problem of the incidence of plane harmonic waves on the interfacial boundary of two joined half-strips with different elastic properties is presented. A detailed analysis is given of the reflection and transmission of the incident wave energy through the interfacial boundary, and the nature of the state of stress and strain is investigated in its neighbourhood. The wave fields in longitudinally inhomogeneous media were studied earlier in /1-3/ etc.

1. We examine an infinite strip of thickness $2h$. We connect it to an x, z Cartesian system of coordinates such that the z axis is orthogonal to the strip boundaries while the x axis coincides with its middle line. Let the plane boundary $x = 0$ be the line separating the properties of the material, and let λ_k, μ_k, ρ_k be the elastic moduli and the density of the material to the left of the interfacial boundary $(x < 0, k = 1)$ and to the right of it $(x > 0, k = 2)$. We shall assume the boundaries of the strip $z = \pm h$ to be stress-free.

We introduce the four-dimensional vector $\mathbf{W} = (u, w, \sigma, \tau)^T$ characterizing the wave field in the strip into the consideration. Here $u = u_x$, $w = u_z$ are the displacement vector components and $\sigma = \sigma_{xx}$, $\tau = \tau_{xz}$ are the corresponding stress tensor components.

Let a plane normal Lamb wave of unit amplitude $\mathbf{W}_j^{(1)}(z, \gamma_j^{(1)}) \exp[i(\gamma_j^{(1)}x - \Omega t)]$ be incident from $x = -\infty$ onto the interfacial boundary, where $\Omega = \omega h/c$ is the dimensionless frequency ($c = \max\{\sqrt{\mu_1/\rho_1}, \sqrt{\mu_2/\rho_2}\}$), $\gamma_j^{(1)}$ is the j-th wave number related to Ω by the Rayleigh-Lamb dispersion equation /4/, and $\mathbf{W}_j^{(1)}$ is a four-dimensional vector whose components are determined for compression-tension waves by the relationships

$$u_j = i\gamma_j(\operatorname{ch}\alpha_1 z - S_j\alpha_2^{-1}\operatorname{ch}\alpha_2 z)$$
$$w_j = \gamma_j^2\alpha_1^{-1}\operatorname{sh}\alpha_1 z - S_j\operatorname{sh}\alpha_2 z$$
$$\tau_j = \mu\left(\frac{\partial u_j}{\partial z} + i\gamma_j w_j\right), \quad \sigma_j = i\gamma_j(\lambda + 2\mu)u_j + \lambda\frac{\partial w_j}{\partial z}$$
$$\alpha_1^2 = \gamma_j^2 - \frac{\rho\Omega^2}{\mu}, \quad \alpha_2^2 = \gamma_j^2 - \frac{\rho\Omega^2}{\lambda + 2\mu}$$
$$S_j = \frac{\alpha_1^2 + \gamma_j^2}{2\alpha_1}\frac{\operatorname{sh}\alpha_1 h}{\operatorname{sh}\alpha_2 h}$$

The superscript $k = 1, 2$ in parentheses indicates that the quantities belong to the medium located, respectively, to the left or the right of the interfacial boundary of the material

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